Multipath Mitigation Techniques for Nonlinear Adaptive Beamforming

Peter Vouras
Naval Research Laboratory, Radar Division
Washington, DC 20375

***Abstract*—Nonlinear adaptive processing using sparse arrays has received a fair amount of attention in the radar community recently because it is possible to exploit more spatial degrees of freedom for beamforming than a conventional linear processor would allow. However, the practical difficulties of nonlinear adaptive beamforming have received little consideration in the literature. This paper will describe the deleterious impact of multipath on the adapted output of a nested array and show that nulling performance is severely degraded. Insight into the multipath problem for nonlinear adaptive beamforming will be derived by analyzing an optimal beamformer which suppresses multipath residue at the adapted output. In addition, an analysis of the effectiveness of using the physical array elements to perform spatial smoothing for mitigating the residual effects of multipath will be presented.**

I. Nonlinear Adaptive Beamforming Using Nested Arrays in the Absence of Multipath

Nested linear arrays are nonuniform passive arrays created by concatenating two or more uniform linear arrays (ULAs) to create an array with increasing inter-sensor spacing. By using the second order statistics of the signals impinging on the array, it is possible to obtain $O(N^2)$ adaptive degrees of freedom from N physical sensors [1]. For example, a two-level nested linear array that is analyzed in this paper is an array of length 12 with 6 elements located at the positions $\{0, 1, 2, 3, 7, 11\}$. The smallest spacing between elements of a nested array corresponds to one-half the wavelength of the highest received frequency to avoid spatial aliasing.

Much of the detailed mathematical background of adaptive beamforming on nested arrays can be found in [1]–[6]. To provide a brief overview of nested array processing consider an N element nonuniform linear array (NULA). Assume M narrowband, uncorrelated signals are arriving at this array from directions $\phi_1, \phi_2, \ldots, \phi_M$ with powers ${\sigma_1}^2, {\sigma_2}^2, \ldots, {\sigma_M}^2$, respectively. Let $\mathbf{v}(\phi)$ be the N-by-1 steering vector corresponding to the direction ϕ,

$$\mathbf{v}(\phi) = \begin{bmatrix} 1 & e^{j(2\pi/\lambda) r_1 \sin\phi} \ldots e^{j(2\pi/\lambda) r_{N-1} \sin\phi} \end{bmatrix}^T \quad (1)$$

where r_i denotes the position of the ith sensor along the array, which is an integer multiple of $\lambda/2$. The received signal at time instant k is

$$\mathbf{x}[k] = \mathbf{A}\mathbf{s}[k] + \mathbf{n}[k] \quad (2)$$

where $\mathbf{A} = [\mathbf{v}(\phi_1) \quad \mathbf{v}(\phi_2) \ldots \mathbf{v}(\phi_M)]$ is the array manifold matrix and $\mathbf{s}[k] = [s_1[k] \quad s_2[k] \ldots s_M[k]]^T$ is the source signal vector. The noise $\mathbf{n}[k]$ is assumed to be spatially white and temporally uncorrelated with the signal sources. Since the signal sources are mutually uncorrelated, the signal covariance matrix $\mathbf{R}_{ss}$ is diagonal and the covariance matrix of the received signal becomes

$$\mathbf{R}_{xx} = E[\mathbf{x}[k]\mathbf{x}[k]^H] = \mathbf{A}\mathbf{R}_{ss}\mathbf{A}^H + {\sigma_n}^2\mathbf{I} \quad (3)$$

$$= \mathbf{A} \begin{bmatrix} {\sigma_1}^2 & 0 & \ldots & 0 \\ 0 & {\sigma_2}^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \ldots & 0 & {\sigma_M}^2 \end{bmatrix} \mathbf{A}^H + {\sigma_n}^2\mathbf{I}. \quad (4)$$

Next the covariance matrix $\mathbf{R}_{xx}$ is vectorized to create the vector

$$\mathbf{z} = \mathrm{vec}(\mathbf{R}_{xx}) = \mathrm{vec}\left[\sum_{i=1}^{M} {\sigma_i}^2 \left(\mathbf{v}(\phi_i)\mathbf{v}(\phi_i)^H\right)\right] + {\sigma_n}^2\mathbf{1}_n \quad (5)$$

$$= (\mathbf{A}^* \odot \mathbf{A})\,\mathbf{p} + {\sigma_n}^2\mathbf{1}_n \quad (6)$$

where * denotes conjugation, $\mathbf{p} = \begin{bmatrix} {\sigma_1}^2 & {\sigma_2}^2 \ldots {\sigma_M}^2 \end{bmatrix}^T$, $\mathbf{1}_n = \begin{bmatrix} {\mathbf{e}_1}^T & {\mathbf{e}_2}^T \ldots {\mathbf{e}_N}^T \end{bmatrix}^T$, and $\mathbf{e}_i$ is a column vector of all zeros except for a one in the ith position. The matrix

$$\mathbf{A}^* \odot \mathbf{A} = \begin{bmatrix} \mathbf{v}(\phi_1)^* \otimes \mathbf{v}(\phi_1) \cdots \mathbf{v}(\phi_M)^* \otimes \mathbf{v}(\phi_M) \end{bmatrix} \quad (7)$$

is the Khatri-Rao product of the matrices $\mathbf{A}^*$ and $\mathbf{A}$ with $\otimes$ denoting the Kronecker product. The vector $\mathbf{z}$ behaves like the received signal at an array whose manifold is given by the matrix $\mathbf{A}^* \odot \mathbf{A}$. The nonlinearly adapted output of the nested array is formed by determining a weight vector $\mathbf{w}$ and computing the inner product $|\mathbf{w}^H\mathbf{z}|$.

II. Impact of Multipath

A central assumption in the development of nested array processing is that the signal sources impinging on the array are temporally uncorrelated so that the signal covariance matrix $\mathbf{R}_{ss}$ is diagonal. In the presence of multipath however this assumption will be violated since delayed replicas of the direct path signals will arrive at the array after being scattered by the environment. In such a scenario, the off-diagonal entries of the signal covariance matrix will be finite and non-negligible and their exact value will depend on the complex correlation coefficient between the direct and scattered signals, which is geometry dependent.

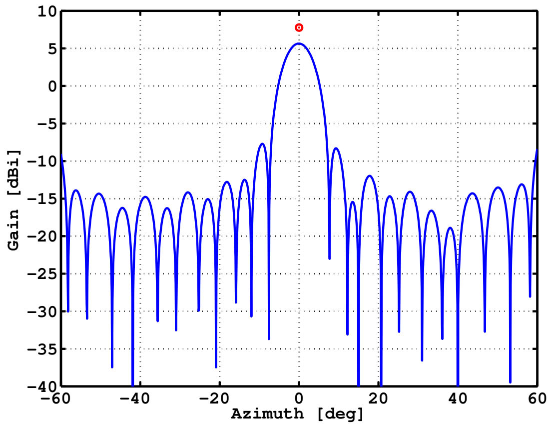

Fig. 1. Multipath Scenario

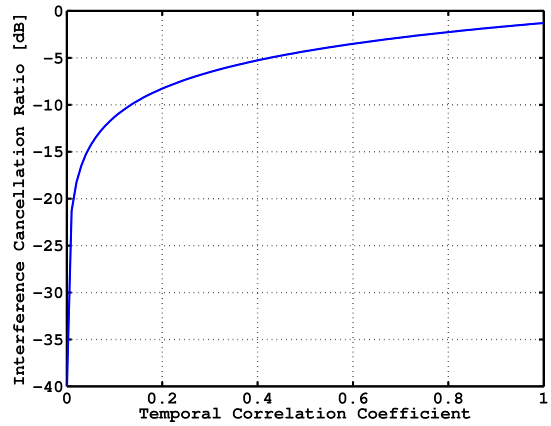

Fig. 2. Cancellation Ratio

Consider the case of a direct path signal arriving through the array mainbeam and a multipath version of the same signal incident in the pattern sidelobes at $15°$. For this case, the signal covariance matrix has the form,

$$\mathbf{R}_{ss} = \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho^*\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}. \tag{8}$$

Suppose a clairvoyant null is placed in the adapted beampattern at exactly $15°$ to cancel the multipath as shown in Fig. 1. The weight vector which forms this beampattern was computed using the projected gradient algorithm described in [7]. The red circle in Fig. 1 corresponds to the gain (7.8 dBi) of the uniformly weighted physical array with six elements. Since the signal covariance matrix $\mathbf{R}_{ss}$ is no longer diagonal, the spatial covariance matrix $\mathbf{R}_{xx}$ in (3) will include outer products of the form $\mathbf{v}(\phi_1)\mathbf{v}(\phi_2)^H$ and $\mathbf{v}(\phi_2)\mathbf{v}(\phi_1)^H$. When $\mathbf{R}_{xx}$ is vectorized and the adapted output created using $\mathbf{w}^H\mathbf{z}$, these outer products will contribute to a residue which cannot be canceled using conventional techniques.

Define the cancellation ratio of the adapted array to be the output residue power divided by the power of the incident multipath. For the case where the direct path signal in the mainbeam is 10 dB, and the multipath signal at $15°$ is 7 dB, Fig. 2 plots the cancellation ratio as a function of the (real) correlation coefficient ρ in (8). For mainbeam and multipath signals with no temporal correlation (i.e, $\rho = 0$), the cancellation ratio is infinite. But, for even small values of ρ the plot shows that cancellation performance quickly degrades.

III. Multipath Suppression Beamformer

Consider a signal covariance matrix $\mathbf{R}_{ss}$ as in (8) for a signal of interest arriving from direction ϕ_1 and a correlated multipath version arriving from direction ϕ_2. The data covariance matrix $\mathbf{R}_{xx}$ is

$$\mathbf{R}_{xx} = \tag{9}$$
$$\begin{bmatrix} \mathbf{v}(\phi_1) & \mathbf{v}(\phi_2) \end{bmatrix} \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho^*\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix} \begin{bmatrix} \mathbf{v}(\phi_1)^H \\ \mathbf{v}(\phi_2)^H \end{bmatrix}$$
$$+\sigma_n^2\mathbf{I}.$$

Define

$$\begin{aligned} \mathbf{a}_{ij} &= \mathbf{v}(\phi_i)^* \otimes \mathbf{v}(\phi_j), \\ \mathbf{p}_i &= \left[\sigma_1\sigma_2\rho^* \quad \sigma_1\sigma_2\rho \quad \sigma_2^2\right]^T, \\ \mathbf{A}_i &= [\mathbf{a}_{12} \quad \mathbf{a}_{21} \quad \mathbf{a}_{22}]. \end{aligned} \tag{10}$$

Then the vectorized covariance matrix $\mathbf{z} = \text{vec}(\mathbf{R}_{xx})$ becomes

$$\begin{aligned} \mathbf{z} &= \sigma_1^2\mathbf{a}_{11} + \mathbf{A}_i\mathbf{p}_i + \sigma_n^2\mathbf{1}_n \\ &= \mathbf{z}_1 + \mathbf{z}_{i+n}. \end{aligned} \tag{11}$$

Consider the problem of reducing the contributions of multipath and noise to the adapted array output while the peak of the mainbeam is constrained to be unity in the desired look direction, ϕ_1. The desired optimization program becomes

$$\begin{aligned} \min \quad & |\mathbf{w}^H\mathbf{z}_{i+n}| \\ \text{such that} \quad & \mathbf{w}^H\mathbf{a}_{11} = 1. \end{aligned} \tag{12}$$

This problem is equivalent to

$$\begin{aligned} \min \quad & \mathbf{w}^H\mathbf{Q}\mathbf{w} \\ \text{such that} \quad & \mathbf{w}^H\mathbf{a}_{11} = 1 \end{aligned} \tag{13}$$

where the rank one matrix $\mathbf{Q} = \mathbf{z}_{i+n}\mathbf{z}_{i+n}^H$. At this point, introduce the diagonal loading factor, $\gamma^2 > 0$, so that the optimization program becomes

$$\begin{aligned} \min \quad & \mathbf{w}^H[\mathbf{Q} + \gamma^2\mathbf{I}]\mathbf{w} \\ \text{such that} \quad & \mathbf{w}^H\mathbf{a}_{11} = 1. \end{aligned} \tag{14}$$

The parameter γ^2 improves regularization by controlling the

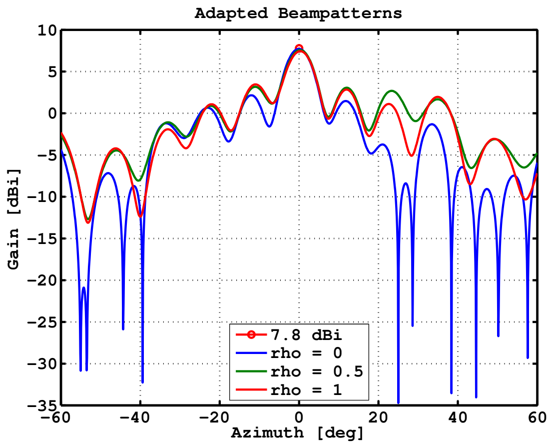

Fig. 3. Adapted Beampatterns: Case 1

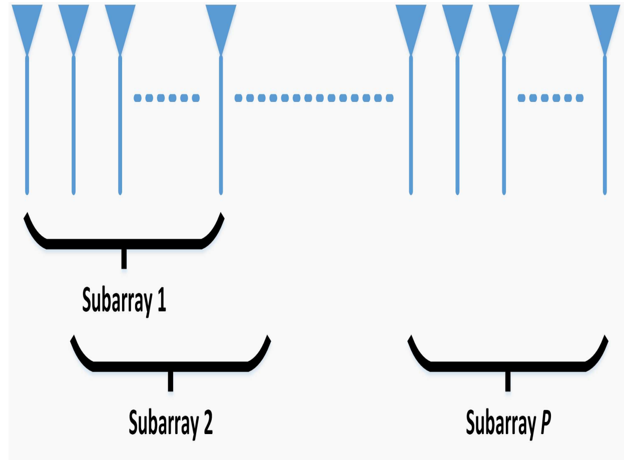

Fig. 4. Array Configuration for Spatial Smoothing

norm of $\mathbf{w}$. The optimal solution to (14) is shown to be

$$\mathbf{w}_{opt} = (1/\delta)[\eta\mathbf{I} - \mathbf{z}_{i+n}\mathbf{z}_{i+n}^H]\mathbf{a}_{11} \tag{15}$$

where

$$\delta = \eta N^2 - |\mathbf{a}_{11}^H\mathbf{z}_{i+n}|^2, \qquad \eta = \gamma^2 + \mathbf{z}_{i+n}^H\mathbf{z}_{i+n}. \tag{16}$$

Figure 3 illustrates the adapted beampatterns for different values of ρ with $\sigma_1 = 100, \sigma_2 = 50, \sigma_n = 1, \gamma^2 = 0.1$ and $\phi_2 = 25°$.

IV. Analysis of Spatial Smoothing Applied to Nested Arrays

The previous beamformer, while optimal for eliminating multipath signals, requires a priori knowledge of $\mathbf{z}_{i+n}$. Another approach to mitigating multipath which does not require any a priori knowledge of the impinging signals is to perform spatial smoothing. This section analyzes the efficacy of spatial smoothing for reducing multipath effects in nonlinear adaptive beamforming on nested arrays. Conventional spatial smoothing partitions a physical array into smaller subarrays and computes the average of the covariance matrices corresponding to each subarray [8]–[10]. This averaging operation decorrelates multipath signals incident on the array and yields a final spatially smoothed covariance matrix which progressively approaches a diagonal matrix. The analysis in this section determines how quickly forward/backward spatial smoothing decorrelates multipath signals when it is applied to nested subarrays formed within a filled ULA.

Consider a filled ULA of length L. The array is divided into $P = L - M + 1$ overlapping nested subarrays of length M with N elements each, as shown in Fig. 4. To determine the rate at which coherent signal sources are decorrelated, consider a scenario with two signal sources. The desired target signal arrives at the array with steering vector $\mathbf{v}(\theta_1)$ and the multipath signal arrives with steering vector $\mathbf{v}(\theta_2)$. The initial correlation of the two signals is ρ. The signal covariance matrix becomes

$$\mathbf{R}_u = \begin{bmatrix} \sigma_1^2 & \sigma_1\sigma_2\rho \\ \sigma_1\sigma_2\rho^* & \sigma_2^2 \end{bmatrix}. \tag{17}$$

The signal correlation coefficient for an arbitrary smoothing index K is shown to be,

$$\rho^{(K)} = \frac{\rho + \rho^* e^{-j\pi(L+1)(\sin\theta_1 - \sin\theta_2)}}{2K} \times e^{-j(K-1)\pi/2(\sin\theta_2 - \sin\theta_1)} \frac{\sin[K\pi/2(\sin\theta_2 - \sin\theta_1)]}{\sin[\pi/2(\sin\theta_2 - \sin\theta_1)]}. \tag{18}$$

This equation shows that for $\theta_1 \approx \theta_2$ and $\rho = \rho^*$, forward-backward smoothing yields no improvement over forward-only smoothing. In general, the performance of spatial smoothing will be geometry and scenario dependent.

A. Nonlinearly Adapted Output After Spatial Smoothing

Given the adaptive weight vector $\mathbf{w}$, this section considers the squared error, $[\eta^2]^{(K)}$, between the adapted array output as $K \to \infty$ (perfectly decorrelated sources) and the adapted output for finite K (sources partially correlated),

$$[\eta^2]^{(K)} = |\mathbf{w}^H\mathbf{z} - \mathbf{w}^H\mathbf{z}^{(K)}|^2. \tag{19}$$

With two signal sources,

$$\mathbf{z}^{(K)} = \text{vec}\left\{ \begin{bmatrix} \mathbf{v}(\theta_1) & \mathbf{v}(\theta_2) \end{bmatrix} \begin{bmatrix} \sigma_1^2 & \sigma_1\sigma_2\rho^{(K)} \\ \sigma_1\sigma_2\rho^{(K)*} & \sigma_2^2 \end{bmatrix} \begin{bmatrix} \mathbf{v}(\theta_1)^H \\ \mathbf{v}(\theta_2)^H \end{bmatrix} + \sigma_n^2\mathbf{I} \right\} = \sigma_1^2\mathbf{a}_{11} + \sigma_2^2\mathbf{a}_{22} + \sigma_n^2\mathbf{1}_n + \sigma_1\sigma_2\rho^{(K)}\mathbf{a}_{12} + \sigma_1\sigma_2\rho^{(K)*}\mathbf{a}_{21}. \tag{20}$$

Also,

$$\mathbf{z} = \text{vec}\left\{ \left[\begin{array}{cc} \mathbf{v}(\theta_1) & \mathbf{v}(\theta_2) \end{array} \right] \left[\begin{array}{cc} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{array} \right] \left[\begin{array}{c} \mathbf{v}(\theta_1)^H \\ \mathbf{v}(\theta_2)^H \end{array} \right] \right. \quad (21)$$
$$\left. + \sigma_n^2 \mathbf{I} \right\} = \sigma_1^2 \mathbf{a}_{11} + \sigma_2^2 \mathbf{a}_{22} + \sigma_n^2 \mathbf{1}_n.$$

Substituting $\mathbf{z}$ and $\mathbf{z}^{(K)}$ into (19) reveals that

$$[\eta^2]^{(K)} = (\sigma_1\sigma_2)^2 \mathbf{w}^H [\mathbf{S}^{(K)} + \mathbf{T}^{(K)}]\mathbf{w} \quad (22)$$

where

$$\mathbf{S}^{(K)} = |\rho^{(K)}|^2 (\mathbf{a}_{12}\mathbf{a}_{12}^H + \mathbf{a}_{21}\mathbf{a}_{21}^H) \quad (23)$$
$$\mathbf{T}^{(K)} = \left[\rho^{(K)}\right]^2 \mathbf{a}_{12}\mathbf{a}_{21}^H + \left[\rho^{(K)}\right]^{2*} \mathbf{a}_{21}\mathbf{a}_{12}^H.$$

V. Simulated Results

A simulation of a filled ULA was used to evaluate the effectiveness of spatial smoothing for mitigating multipath in nonlinear adaptive beamforming. A desired target signal and a scattered multipath signal were generated at each array element with a phase shift determined by the path length from the array element to the signal source. Spatial smoothing was applied over K steps using overlapping nested subarrays. The final spatially smoothed covariance matrix was vectorized and the inner product with a weight vector was computed to form the adapted array output.

Figure 5 illustrates the magnitude of the correlation coefficient between the target signal and multipath as a function of K. It is assumed that the initial correlation $\rho_0 = e^{j\pi/4}$. Results are compared for forward-only smoothing and forward-backward smoothing for an angular source separation of $\Delta\theta = 10°$. In one configuration, the sources are at $10°, 20°$ and in the second case they are at $65°, 75°$. The plot shows that when the 2 sources are closer to end-fire, they require more spatial smoothing steps to decorrelate. Also, there is a very clear performance improvement due to forward-backward smoothing as opposed to forward-only smoothing. Figure 6 illustrates the squared error of the array output as defined in (19) for $\Delta\theta = 20°$ and $\rho_0 = e^{j\pi/4}$.

Figure 7 illustrates an ideal adapted beampattern with a desired null at $20°$. Figure 8 plots the adapted beampattern with forward-backward smoothing for $K = 1, 11, 43$. The target SNR at each array element is 60 dB and the multipath INR is 26 dB. Figure 9 shows the case with INR equal to 56 dB. In this scenario, many more spatial smoothing steps, and a longer array, are required to decorrelate the signals and restore the adapted pattern. In particular, after $K = 43$ steps, forward-backward spatial smoothing restores the desired null at $20°$.

VI. Conclusions

This paper describes the impact of multipath on the nonlinearly adapted output of a sparse nested array. It is shown that for even slight correlation between the direct path signal incident on the array and a scattered multipath version

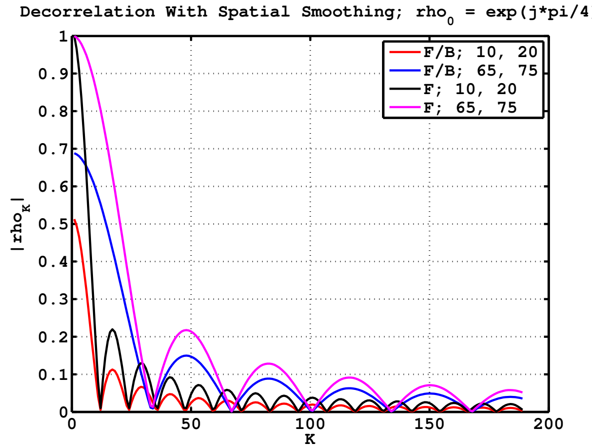

Fig. 5. Decorrelation Rate; $\rho_0 = e^{j\pi/4}, \Delta\theta = 10°$

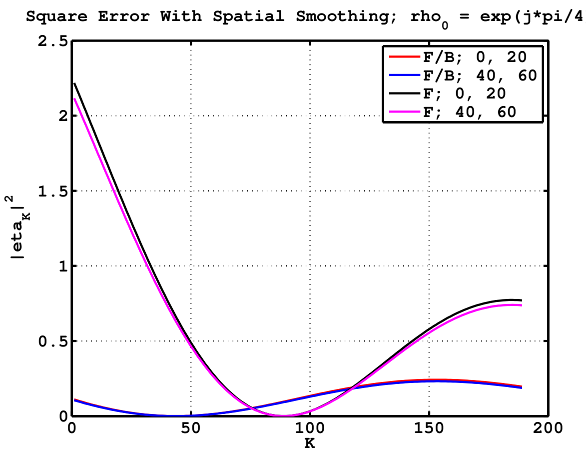

Fig. 6. Squared Error; $\rho_0 = e^{j\pi/4}, \Delta\theta = 20°$

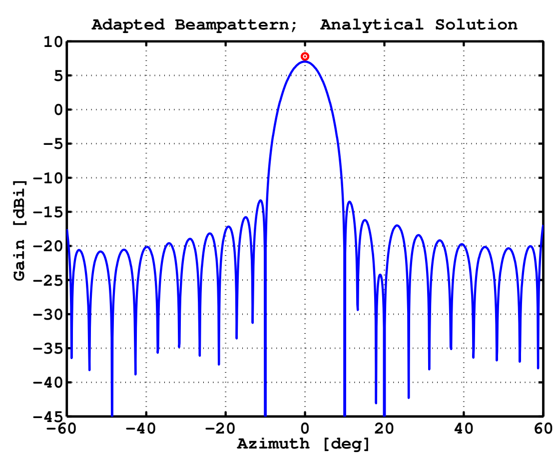

Fig. 7. Ideal Adapted Pattern; Desired Null at $20°$

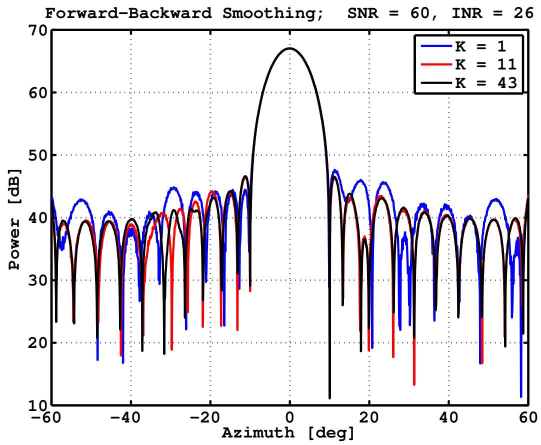

Fig. 8. Forward-Backward Smoothing; SNR = 60 dB, INR = 26 dB

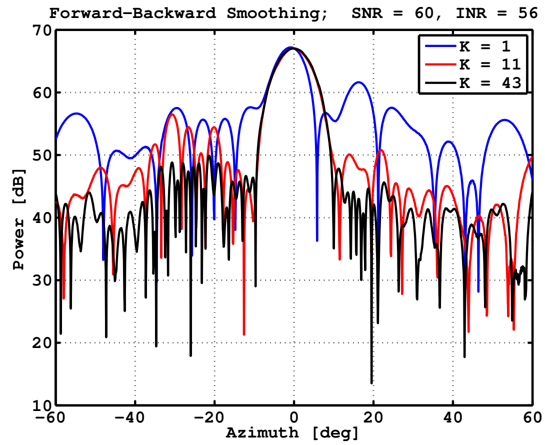

Fig. 9. Forward-Backward Smoothing; SNR = 60 dB, INR = 56 dB

entering through the sidelobes, the cancellation performance of the array quickly degrades. Spatial smoothing applied to the physical array elements may decorrelate the target signal and multipath before any nonlinear processing occurs but its effectiveness is dependent on geometry and the relative strength of the target and multipath signals.

VII. Acknowledgements

This work was carried out under the Naval Research Laboratory Base Program sponsored by the Office of Naval Research.

References

[1] P. Pal and P. P. Vaidyanathan, "Nested Arrays: A Novel Approach to Array Processing with Enhanced Degrees of Freedom," *IEEE Transactions on Signal Processing*, Vol. 58, No. 8, August 2010.

[2] W. Ma, T. Hsieh, and C. Chi, "DOA Estimation of Quasi-Stationary Signals via Khatri-Rao Subspace," Proceedings of the International Conference on Acoustics, Speech, and Signal Processing (ICASSP), Taipei, Taiwan, April 19-24, 2009.

[3] R. T. Hoctor and S. Kassam, "The Unifying Role of the Coarray in Aperture Synthesis for Coherent and Incoherent Imaging," *Proceedings of the IEEE*, Volume 78, Issue 4, 1990.

[4] P. Pal and P. P. Vaidyanathan, "Nested Arrays in Two Dimensions, Part I: Geometrical Considerations," *IEEE Transactions on Signal Processing*, Vol. 60, No. 9, September 2012.

[5] P. Pal and P. P. Vaidyanathan, "Nested Arrays in Two Dimensions, Part II: Application in Two Dimensional Array Processing," *IEEE Transactions on Signal Processing*, Vol. 60, No. 9, September 2012.

[6] P. Pal and P. P. Vaidyanathan, "Efficient Frequency Invariant Beamforming Using Virtual Arrays," Proceedings of the Forty Fourth Asilomar Conference on Signals, Systems, and Computers, Pacific Grove, CA., November 7-10, 2010.

[7] P. Vouras, "Sample Support Requirements for Nonlinear Adaptive Beamforming," Proceedings of the Conference on Information Sciences and Systems, Baltimore, MD., March 18-20, 2015.

[8] V. U. Reddy, A. Paulraj, and T. Kailath, "Performance Analysis of the Optimum Beamformer in the Presence of Correlated Sources and Its Behavior Under Spatial Smoothing," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, Vol. ASSP-35, No. 7, July 1987.

[9] S. U. Pillai, B. H. Kwon, "Forward/Backward Spatial Smoothing Techniques for Coherent Signal Identification," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, Vol. 37, No. 1, January 1989.

[10] Y. Chen, "On Spatial Smoothing for Two-Dimensional Direction-of-Arrival Estimation of Coherent Signals," *IEEE Transactions on Signal Processing*, Vol. 45, No. 7, July 1997.