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Robust Transmit Nulling in Phased Array Antennas

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***Abstract*— The ability to create nulls in the transmit pattern of a phased array antenna has many applications for communication and radar systems, including interference and clutter mitigation. Most nulling techniques introduce small perturbations in amplitude and phase, or phase-only, at each element of the phased array. For ease of implementation, phase-only perturbations are usually desired and provide acceptable null depths. However, the phase shift at each array element will vary with the frequency of the transmitted signal. As a result, the depth and pointing accuracy of the transmit null will not be uniform over the bandwidth of the transmitted signal. A more robust transmit nulling approach is to insert a tapped delay line (TDL) behind each array element instead of a phase shifter. A significant drawback to this approach is that it is difficult to implement due to the high signal sampling frequency required. To reduce the implementation complexity of the TDL architecture, a filter bank is introduced behind each array element. The proposed architecture partitions the transmit signal into independent subbands and inserts a TDL in each subband to form a wideband spatial null. When the coefficients of the TDLs are subject to quantization errors, a novel band partitioning scheme based on Principal Component Filter Banks (PCFBs) is shown to yield superior performance as compared to more conventional filter banks, such as the Discrete Fourier Transform Filter Bank (DFTFB).**

I. Problem Description

An important capability in modern phased array systems is the ability to place nulls in the antenna transmit pattern. Transmit nulls are useful for mitigating interference in dense operating environments and for reducing the unwanted backscatter from clutter. In most practical systems, amplitude and phase control is not available at the array element level. Instead, independent phase commands are applied at each array element to form the desired spatial null, with the amplitude weight at each element fixed to unity. Some common techniques for computing transmit nulls are described in [1] – [4]. These techniques are limited however in that the formed spatial null is maintained only over a narrow signal bandwidth. An array architecture for forming wideband spatial nulls is shown in Fig. 1.

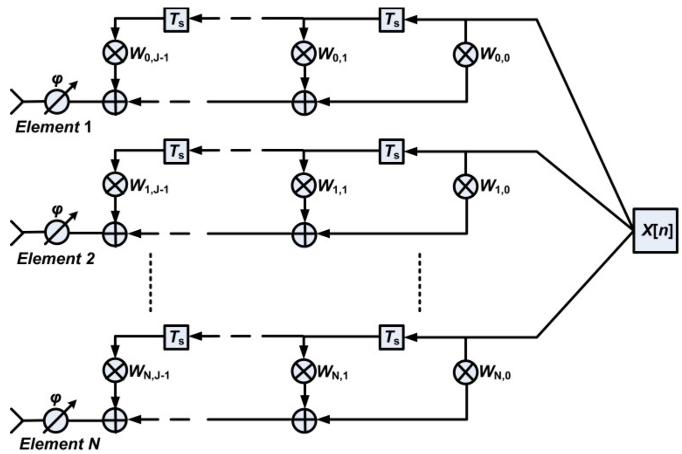

Figure 1. Wideband Array Architecture

II. Tapped Delay Line Architecture for Wideband Nulls

Figure 1 illustrates the proposed array architecture for forming wideband transmit nulls. Behind each array element is a Tapped Delay Line (TDL) with J real taps spaced T_s seconds apart. The impulse response of a TDL array with N elements can be written as

$$H(\omega,\theta)=\sum_{m=0}^{J-1} e^{-jm\omega T_s}\sum_{n=0}^{N-1} w_{nm} e^{-jn\varphi} \quad (1)$$

where w_{nm} is the mth tap coefficient of the nth array element, and φ is the phase difference between adjacent array elements given by

$$\varphi=\frac{2\pi D\sin\theta}{\lambda}. \quad (2)$$

The parameter D is equal to the distance between array elements, λ is the wavelength of the transmitted signal, and θ is the main beam steering direction with respect to array normal. If the highest transmit frequency of interest is f_1, then D is chosen to be $\lambda_1/2$ to avoid spatial aliasing and T_s is chosen to be $1/(2f_1)$ to avoid temporal aliasing. A major drawback with the TDL architecture is the high sampling frequency associated with the delay between filter taps.

III. BAND PARTITIONING

A difficulty with implementing the TDL array architecture is the high sampling frequency f_s associated with the inter-tap delays. To reduce the implementation complexity of the TDL array architecture, the transmitted signal may be partitioned into M independent subbands using a filter bank behind each element. With a uniform (or maximally decimated) filter bank, the sampling rate in each subband is decimated by a factor of M, which allows for a more practical implementation. The desired filter bank for this application is a paraunitary filter bank which preserves the energy content of the input signal.

A. Discrete Fourier Transform Filter Bank

One of the simplest and most common paraunitary filter banks is the Discrete Fourier Transform Filter Bank (DFTFB). The DFTFB is a FIR perfect reconstruction system with a bank of M analysis filters that are uniformly shifted versions of a lowpass prototype filter $H_0(z)$ [6]. In other words,

$$H_k(z) = H_0\left(zW^k\right), \tag{3}$$

where $W = e^{-j2\pi/M}$ and

$$H_0(z) = 1 + z^{-1} + \cdots + z^{-(M-1)}. \tag{4}$$

Notice that the filters have length M, which is equal to the number of filter bank channels. The analysis polyphase matrix $\mathbf{E}(z) = \mathbf{W}^*$, where $\mathbf{W}$ is the M-by-M DFT matrix with elements $[\mathbf{W}]_{km} = W^{km}$. The synthesis polyphase matrix $\mathbf{R}(z) = \mathbf{W}$, and the reconstructed signal at the output of the filter bank is

$$\hat{x}[n] = Mx[n - M + 1], \tag{5}$$

which is simply a delayed version of the input. The synthesis filters are related by

$$F_k(z) = W^{-k} F_0\left(zW^k\right) \tag{6}$$

where $F_0(z) = H_0(z)$. Thus each synthesis filter has the same magnitude response as the corresponding analysis filter.

B. Band Partitioned Tranmsit Nulling Algorithm

To compute the TDL coefficients for a wideband null, the Signal-to-Noise-plus-Interference Ratio (SINR) integrated over the signal bandwidth of interest for a hypothetical interference source in the direction of the desired null is maximized. This objective function is derived as follows.

Define a frequency-dependent steering vector for a linear array with N elements in the direction θ as

$$\mathbf{v}(\theta, f) = \left[v_1(\theta, f), v_2(\theta, f), \ldots, v_N(\theta, f)\right], \tag{7}$$

where each component in (7) is given by

$$v_k(\theta, f) = \exp\left(j2\pi D \tfrac{f}{c} \sin\theta(k-1)\right), \qquad k = 1, \ldots, N \tag{8}$$

with D equal to the inter-element spacing, f the transmit signal frequency, θ the steering angle of the signal with respect to array normal, and c the speed of light.

A vector $\mathbf{w}$ of frequency-dependent weights to be applied to each array element is defined as

$$\mathbf{w}(f) = \left[w_1(f), w_2(f), \ldots, w_N(f)\right]^T \tag{9}$$

where $w_j(f)$ is the frequency response of the jth TDL $h_j[n]$ and

$$w_j(f) = H_j(f) = \sum_{k=0}^{J-1} h_j[k] e^{-j\omega T_s k}. \tag{10}$$

The frequency-dependent signal $\mathbf{R}_{vv}$ and noise $\mathbf{N}_{vv}$ covariance matrices can be written for the main beam steering direction θ and the direction θ_1 of the desired null as

$$\mathbf{R}_{vv}(f) = \mathbf{v}(\theta, f)\mathbf{v}(\theta, f)^H, \tag{11}$$

$$\mathbf{N}_{vv}(f) = \beta\mathbf{v}(\theta_1, f)\mathbf{v}(\theta_1, f)^H + \sigma^2\mathbf{I} \tag{12}$$

where σ^2 is the power of a zero-mean additive white noise Gaussian process, and β is a real positive scalar. The cost function to be maximized over the signal bandwidth of interest is the integrated SINR

$$\xi = \int_{f_0}^{f_1} \frac{\mathbf{w}^H \mathbf{R}_{vv} \mathbf{w}}{\mathbf{w}^H \mathbf{N}_{vv} \mathbf{w}} W(f)\, df \tag{13}$$

where f_0 and f_1 are the frequency endpoints of the signal bandwidth and $W(f)$ is a scalar nonnegative real weighting function. The metric used to gauge nulling performance is the average null depth d over the bandwidth of interest, defined as

$$d = \frac{1}{P}\sum_{k=0}^{P-1} \left|G(f_k, \theta_1)\right|^2, \tag{14}$$

where P is the number of frequency samples within the signal bandwidth, θ_1 is the desired null location, and $G(f, \theta_1)$ is the frequency-dependent array voltage pattern. For the case where M_n nulls in the array pattern are desired, the noise covariance matrix is written as

$$\mathbf{N}_{vv}(f) = \beta\sum_{j=1}^{M_n} \mathbf{v}(\theta_j, f)\mathbf{v}(\theta_j, f)^H + \sigma^2\mathbf{I}. \tag{15}$$

For a band partitioned array architecture, the optimization objective must be maximized independently for each subband, and the scalar weighting function $W(f)$ in (13) is set equal to the magnitude squared response of each analysis filter. Thus, the objective function to be maximized in the jth subband becomes

$$\xi_j = \int_{f_0}^{f_1} \frac{\mathbf{w}_j^H \mathbf{R}_{vv} \mathbf{w}_j}{\mathbf{w}_j^H \mathbf{N}_{vv} \mathbf{w}_j} \left| H_j(f) \right|^2 df \ . \tag{16}$$

This objective may be maximized using the conjugate gradient algorithm described in [7].

C. *Simulated Results*

Figures 2 and 3 illustrate the array pattern for a point null at 19.57º for a fractional bandwidth of 40% using a DFTFB. The TDL in each subband has 3 taps. The average null depth is −96.0 dBi.

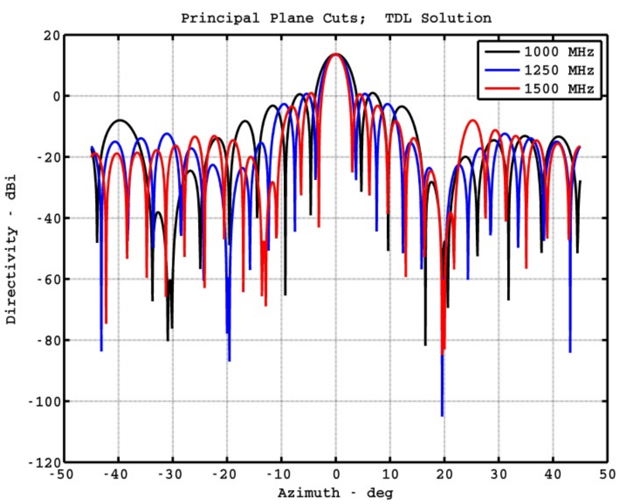

Figure 2. Point Null – DFTFB Solution

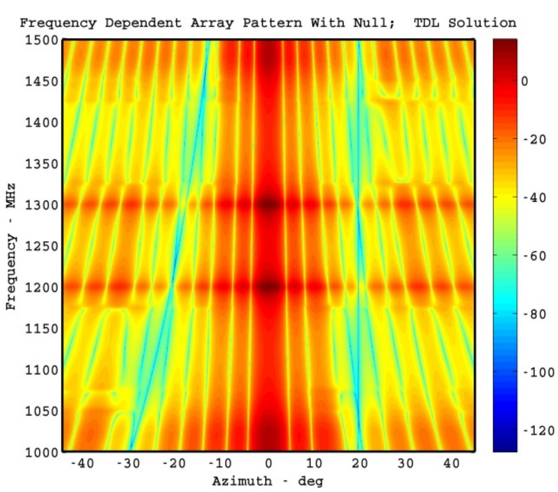

Figure 3. Array Pattern – Point Null DFTFB Solution

IV. Transmit Signal Distortion

The main beam signal transmitted by the array will be the coherent sum in space of the signal radiated by each antenna element. With the band partitioned architecture however, the transmit signal at each array element will be slightly different at the output of each filter bank, which may result in a distorted signal in the main beam. Furthermore, the extent of signal distortion will depend on the TDL coefficients and therefore may vary with each different null solution. To compute the distortion introduced in the spectrum of the main beam signal by the band partitioned architecture, the signal radiated into space by each array element must be calculated.

Let $Y_n(e^{j\omega})$ denote the signal radiated by the nth array element and $S_n(e^{j\omega})$ the signal at the output of the nth filter bank. Then,

$$Y_n\left(e^{j\omega}\right) = S_n\left(e^{j\omega}\right) e^{-j2\pi n (D/\lambda)\sin\theta} \tag{17}$$

for $0 \le n \le N-1$. If $X(e^{j\omega})$ denotes the signal at the input to each filter bank, $H_k(e^{j\omega})$ the response of the kth analysis filter, $F_k(e^{j\omega})$ the response of the kth synthesis filter, and $T_k(e^{j\omega})$ the response of the kth TDL, all at the nth array element, then

$$S_n\left(e^{j\omega}\right) = \frac{1}{M} \sum_{l=0}^{M-1} X\left(e^{j(\omega - 2\pi l/M)}\right) \sum_{k=0}^{M-1} H_k\left(e^{j(\omega - 2\pi l/M)}\right) T_k\left(e^{j\omega M}\right) F_k\left(e^{j\omega}\right). \tag{18}$$

The main beam signal can be computed as,

$$A\left(e^{j\omega}\right) = \sum_{n=0}^{N-1} Y_n\left(e^{j\omega}\right) \tag{19}$$

and compared to $X(e^{j\omega})$ to determine the induced signal distortion. For the point null DFTFB solution, the main beam signal distortion created by the different TDL phase and magnitude responses is shown in Fig. 4 which overlays $X(e^{j\omega})$ and $A(e^{j\omega})$. The blue curve shows the spectrum of the intended transmit signal $X(e^{j\omega})$. This spectrum corresponds to a general autoregressive signal and does not represent any particular radar waveform.

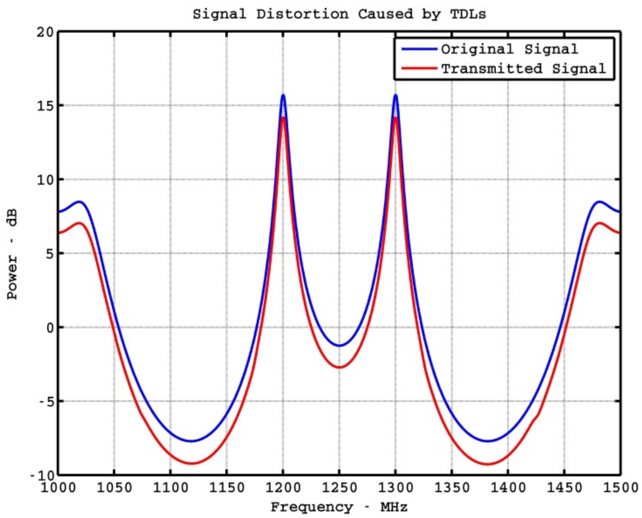

Figure 4. Main Beam Signal Distortion – Point Null Solution

V. TDL Coefficient Quantization

For real time transmit nulling operation in any practical radar system with hundreds or perhaps thousands of elements, it is not feasible to recompute the coefficients of the TDLs behind each array element for every new null direction or main beam steering angle. Instead, the TDL coefficients must be pre-computed and retrieved from a look-up table. The resulting number of data points in the table could be substantial if every possible combination of main beam and null steering directions is accounted for. To reduce the memory footprint of such a scheme, the TDL coefficients must be quantized and stored in fixed precision using as few bits as possible. The effect of such an implementation on nulling performance is to introduce errors in the frequency response of each TDL. These errors in turn will degrade the depth of the transmit null. Quantizing the TDL coefficients can be modeled by adding random quantization noise to each infinite precision coefficient.

Assume a B-bit quantizer is used to represent the computed TDL coefficients in finite precision. The output of the quantizer can be modeled by adding random errors $e[n]$ to the real TDL coefficients $h[n]$. If $-1 \le h[n] \le 1$, then the smallest quanta at the output of the quantizer is $1/2^B$. The quantization errors will be uniformly distributed in the interval $[-1/2^{B+1}, 1/2^{B+1}]$ with a variance equal to $2^{-2B}/12$.

VI. Quantization Effects on TDL Frequency Response

If the TDL coefficients in each subband are stored in fixed precision, the TDL frequency response will differ from ideal. It is possible to derive an upper bound for the error in the magnitude squared response of a TDL due to coefficient quantization. Let $D(\theta)$ denote a desired TDL frequency response, where $0 \le \theta \le 2\pi$ is normalized frequency, and assume a direct realization of the TDL using coefficients $\mathbf{b} = [b_0, b_1, \ldots, b_{J-1}]^{\mathbf{T}}$. The quantized parameters $\mathbf{p} = [p_0, p_1, \ldots, p_{J-1}]^{\mathbf{T}}$ are obtained by adding quantization errors e_k to the ideal coefficients as in

$$p_k = b_k + e_k\,, \qquad k = 0,1,\ldots,J-1. \tag{20}$$

Let $T(\theta)$ represent the frequency response of the quantized TDL. Then using a Taylor Series Expansion yields,

$$|T(\theta)|^2 \approx |D(\theta)|^2 + \nabla_{\mathbf{b}}\left(|D(\theta)|^2\right)^T(\mathbf{p}-\mathbf{b}) \tag{21}$$

where

$$D(\theta) = \sum_{k=0}^{J-1} b_k e^{-j\theta k}, \quad T(\theta) = \sum_{k=0}^{J-1} p_k e^{-j\theta k}. \tag{22}$$

Thus,

$$|T(\theta)|^2 - |D(\theta)|^2 \approx \sum_{k=0}^{J-1} \frac{\partial |D(\theta)|^2}{\partial b_k}(p_k - b_k). \tag{23}$$

The derivative formulas

$$\frac{\partial D(\theta)}{\partial b_k} = e^{-j\theta k}, \quad \frac{\partial T(\theta)}{\partial p_k} = e^{-j\theta k} \tag{24}$$

and the product rule for differentiation yield,

$$\frac{\partial |D(\theta)|^2}{\partial b_k} = 2\,\mathrm{Re}\left\{e^{j\theta k} D(\theta)\right\}. \tag{25}$$

Let L denote the full scale value used for the coefficient b_k. One way of choosing the full scale value is to find the coefficient having the largest magnitude from all the TDLs and use it as the full scale value for all the coefficients [8]. In other words,

$$L = \max\left\{|b_{kjl}|, 0 \le k \le J-1, 0 \le j \le M-1, 0 \le l \le N-1\right\} \tag{26}$$

where the index k varies across all TDL taps, j varies across all subbands, and l varies across all array elements. Then,

$$|p_k - b_k| \le Lq \tag{27}$$

where $q = 2^{-B}$ and B is the number of quantizer bits. Thus,

$$\left||T(\theta)|^2 - |D(\theta)|^2\right| \le q \sum_{k=0}^{J-1}\left|\frac{\partial |D(\theta)|^2}{\partial b_k}\right| L \le 2^{-B+1}\sum_{k=0}^{J-1}|D(\theta)|L\,. \tag{28}$$

Since any TDL is stable and has finite coefficients, there exists a finite constant D_{MAX} such that

$$\left|D(\theta)\right| \le \sum_{k=0}^{J-1}\left|b_k\right| \le D_{MAX} \tag{29}$$

for any TDL in any subband of any array element. Thus,

$$\left|\left|T(\theta)\right|^2 - \left|D(\theta)\right|^2\right| \le 2^{-B+1}\sum_{k=0}^{J-1} D_{MAX}L \le 2^{-B+1}\Phi\ . \tag{30}$$

where $\Phi = LJD_{MAX}$.

Let the input v_j to the TDL $\mathbf{p}_j$ in the jth subband be a zero mean wide sense stationary (WSS) process with power spectral density (psd) $S_{vv}(\theta)$. Define the error term $\varepsilon_j(\theta)$ at the output of the TDL to be

$$\varepsilon_j(\theta) = S_{vv}(\theta)\left(\left|T(\theta)\right|^2 - \left|D(\theta)\right|^2\right). \tag{31}$$

An upper bound for $\varepsilon_j(\theta)$ is

$$\varepsilon_j(\theta) \le S_{vv}(\theta)\left|\left|T(\theta)\right|^2 - \left|D(\theta)\right|^2\right| \le S_{vv}(\theta)2^{-B+1}\Phi\ . \tag{32}$$

Note that this upper bound holds for all the TDLs in the array and that the constant Φ can be taken to be independent of the band partitioning scheme in any filter bank. Integrating $\varepsilon_j(\theta)$ over all frequencies yields

$$\psi_j = \frac{1}{2\pi}\int_0^{2\pi}\varepsilon_j(\theta)d\theta \le \Phi 2^{-B+1}\sigma_j^2 \tag{33}$$

where the subband variance σ_j^2 is equal to

$$\sigma_j^2 = \frac{1}{2\pi}\int_0^{2\pi} S_{vv}(\theta)d\theta\ . \tag{34}$$

The impact of TDL quantization on null depth can be mitigated by minimizing the sum of ψ_j over all M subbands. Since the factor 2Φ does not affect the minimization, it may be dropped. Thus the criterion for minimizing TDL quantization effects becomes a matter of choosing a filter bank from the class of paraunitary filter banks that minimizes the objective function

$$\eta = \sum_{j=0}^{M-1} 2^{-B_j}\sigma_j^2\ , \tag{35}$$

where B_j is the number of quantization bits allocated to the jth subband. Notice the minimization objective is purely a function of the subband variances. The optimal filter bank for this problem is described in the next section.

VII. PRINCIPAL COMPONENT FILTER BANKS

A. *Definition*

Principal Component Filter Banks (PCFBs) have been studied extensively in the literature and have been found to be optimal for a variety of applications, including maximizing the coding gain in data compression applications [9 – 10]. In this section, a proof will be provided to show that PCFBs are also optimal when the subband processing in an orthonormal filter bank includes a quantized TDL and the signal errors at the output of the filter bank must be minimized. In some sense, PCFBs are filter banks that have been adapted to the characteristics of the input signal and can be thought of as a generalized eigendecomposition.

Let σ_j^2 denote the variance of the jth subband signal produced by feeding the scalar signal $x[n]$ to the input of the filter bank. To every possible filter bank within the class of paraunitary filter banks, there corresponds a subband variance vector $\boldsymbol{\sigma}^2$. Let $\mathbf{x}[n]$ denote the M-fold blocked version of $x[n]$ as follows

$$\mathbf{x}[n] = [\,x[n],\quad x[n-1],\quad \cdots\quad x[n-M+1]\,]^T\ . \tag{36}$$

The vector $\mathbf{x}[n]$ is a WSS process with psd matrix $\mathbf{S}_{xx}(e^{j\omega})$. The subband variance vector associated with $x[n]$ is defined as the vector

$$\boldsymbol{\sigma}^2 = \left[\sigma_0^2,\quad \sigma_1^2,\quad \cdots\quad \sigma_{M-1}^2\right]^T\ . \tag{37}$$

Given the filter bank analysis polyphase matrix $\mathbf{E}(z)$ and the psd matrix $\mathbf{S}_{xx}(e^{j\omega})$ which depends on the input signal, the subband variance vector can be computed as

$$\boldsymbol{\sigma}^2 = \frac{1}{2\pi}\int_0^{2\pi}\mathrm{diag}\left(\mathbf{E}\left(e^{j\omega}\right)\mathbf{S}_{xx}\left(e^{j\omega}\right)\mathbf{E}^H\left(e^{j\omega}\right)\right)d\omega\ . \tag{38}$$

The optimization search space for the objective function in (35) is defined to be the set of all subband variance vectors corresponding to all filter banks in the class C of uniform orthonormal filter banks. A filter bank in C is said to be a PCFB for C, and $\mathbf{S}_{xx}(e^{j\omega})$, if its subband variance vector majorizes the subband variance vector of every other filter bank in C.

B. *Optimality*

Majorization is a crucial property of PCFBs. Let $A = \{a_0, a_1, \ldots, a_{M-1}\}$ and $B = \{b_0, b_1, \ldots, b_{M-1}\}$ be two sets having M real numbers. The set A is said majorize the set B if the elements of these sets, when ordered such that $a_0 \ge a_1 \ge \ldots \ge a_{M-1}$ and $b_0 \ge b_1 \ge \ldots \ge b_{M-1}$, obey the property that

$$\sum_{j=0}^{P} a_j \ge \sum_{j=0}^{P} b_j \tag{39}$$

for all $P = 0, 1, \ldots, M-1$ with equality holding when $P = M-1$. Given two real valued vectors $\mathbf{a}$ and $\mathbf{b}$, $\mathbf{a}$ is said to majorize

b when the set of entries of **a** majorizes that of **b**. Any permutation of **a** will also majorize any permutation of **b**.

A useful fact for real vectors **a** and **b** is that the statement **a** majorizes **b** is equivalent to saying there exists an orthostochastic matrix **Q** (corresponding to some unitary matrix **U**) such that **b** = **Qa**. An orthostochastic matrix **Q** is one that can be obtained from a unitary matrix **U** by replacing each entry u_{ij} by $q_{ij} = |u_{ij}|^2$ [11]. The majorization property of PCFBs can be used to prove the following important claim.

Claim: Among all orthonormal filter banks, the filter bank which minimizes the objective function η given in (35) is a PCFB.

Proof: Label the filter bank channels such that $\sigma_0^2 \geq \sigma_1^2 \geq \ldots \geq \sigma_{M-1}^2$. Next allocate the quantization bits B_j to each channel such that $B_0 \geq B_1 \geq \ldots \geq B_{M-1}$. In essence, allocate more bits and greater resolution to the channels with greater signal energy. Observe the objective function η can be rewritten as

$$\eta = \sum_{j=0}^{M-2} \left(2^{-B_j} - 2^{-B_{j+1}}\right)\left(\sum_{k=0}^{j} \sigma_k^2\right) + 2^{-B_{M-1}}\left(\sum_{k=0}^{M-1} \sigma_k^2\right). \quad (40)$$

The last term is constant for all filter banks and can be ignored. Since

$$2^{-B_j} - 2^{-B_{j+1}} \leq 0\,, \quad (41)$$

η is minimized by a PCFB, which by the majorization property maximizes all the partial sums

$$\sum_{k=0}^{j} \sigma_k^2 \quad (42)$$

for $j = 0, 1, \ldots, M-1$.

C. Construction of Ideal PCFB

The class C of uniform orthonormal filter banks includes both FIR filter banks and ideal unrealizable filter banks of infinite order. One may think of C as the set of all $M \times M$ matrices $\mathbf{E}(e^{j\omega})$ that are unitary for all ω. The first step to designing a PCFB is to construct the desired polyphase matrix $\mathbf{D}(e^{j\omega})$.

Given a signal $x[n]$, let $\mathbf{D}(e^{j\omega})$ in the class of orthonormal polyphase matrices diagonalize the psd matrix $\mathbf{S}_{xx}(e^{j\omega})$ for each ω; that is,

$$\mathbf{D}\left(e^{j\omega}\right)\mathbf{S}_{xx}\left(e^{j\omega}\right)\mathbf{D}^{H}\left(e^{j\omega}\right) = \mathbf{\Lambda}\left(e^{j\omega}\right) \quad (43)$$

where $\mathbf{\Lambda}(e^{j\omega})$ is a diagonal matrix for all ω with diagonal entries equal to $(\lambda_0(e^{j\omega}), \lambda_1(e^{j\omega}), \ldots, \lambda_{M-1}(e^{j\omega}))^{\mathbf{T}} = \boldsymbol{\lambda}$. The subband variance vector **v** of an arbitrary filter bank in C is given by

$$\begin{aligned} 2\pi\mathbf{v} &= \int_0^{2\pi} \operatorname{diag}\left(\mathbf{E}\left(e^{j\omega}\right)\mathbf{S}_{xx}\left(e^{j\omega}\right)\mathbf{E}^{H}\left(e^{j\omega}\right)\right) d\omega \\ &= \int_0^{2\pi} \mathbf{Q}\left(e^{j\omega}\right)\left[\lambda_0\left(e^{j\omega}\right), \lambda_1\left(e^{j\omega}\right), \ldots, \lambda_{M-1}\left(e^{j\omega}\right)\right]^T d\omega. \end{aligned} \quad (44)$$

Note that at each frequency ω, the matrix $\mathbf{Q}(e^{j\omega})$ is an orthostochastic matrix corresponding to the unitary matrix $\mathbf{E}(e^{j\omega})\mathbf{D}^H(e^{j\omega})$. Thus at every frequency ω, the integrand vector $\boldsymbol{\lambda}$ of (44) produced by the filter bank $\mathbf{D}(e^{j\omega})$ in C majorizes the corresponding vector **v** of any other filter bank in C. This relation holds no matter how the eigenvalues $\lambda_j(e^{j\omega})$ are ordered in (44). The integration process preserves the majorization relation, if and only if, the $\lambda_j(e^{j\omega})$ are ordered such that $\lambda_0(e^{j\omega}) \geq \lambda_1(e^{j\omega}) \geq \ldots \geq \lambda_{M-1}(e^{j\omega})$ for all ω. This ordering is called spectral majorization. Thus, a filter bank $\mathbf{D}(e^{j\omega})$ in C is also a PCFB, if and only if, it totally decorrelates the subband processes (which is equivalent to diagonalizing the psd matrix $\mathbf{S}_{xx}(e^{j\omega})$) and it causes spectral majorization. In short, the desired polyphase matrix $\mathbf{D}(e^{j\omega})$ of a PCFB can be constructed by performing an eigendecomposition of $\mathbf{S}_{xx}(e^{j\omega})$ at every frequency ω and by setting $\mathbf{D}(e^{j\omega})$ equal to the unitary matrix of eigenvectors for each ω [11].

D. Minimum Mean Square Error Design

Once the desired polyphase matrix $\mathbf{D}(e^{j\omega})$ has been constructed, it may be unrealizable because it has ideal brickwall channel filters with infinite order. Therefore, given $\mathbf{D}(e^{j\omega})$, a FIR paraunitary filter bank $\mathbf{H}(e^{j\omega})$ must be derived to approximate it. This filter bank may be designed by minimizing the mean square error (MSE)

$$\rho = \frac{1}{2\pi}\int_0^{2\pi} \left\|\mathbf{D}\left(e^{j\omega}\right) - \mathbf{H}\left(e^{j\omega}\right)\right\|_2^2 d\omega\,. \quad (45)$$

To solve this problem using unconstrained minimization techniques, the paraunitary condition may be embedded in the objective function by decomposing $\mathbf{H}(e^{j\omega})$ into a product of unitary matrices $\mathbf{R}_j$ using the factorization [6]

$$\mathbf{H}(z) = \mathbf{R}_N \mathbf{\Lambda}(z)\mathbf{R}_{N-1}\mathbf{\Lambda}(z)\cdots\mathbf{\Lambda}(z)\mathbf{R}_0\,. \quad (46)$$

Here $\mathbf{\Lambda}(z)$ is the delay chain matrix

$$\mathbf{\Lambda}(z) = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & z^{-1}\mathbf{I} \end{bmatrix}. \quad (47)$$

Since any unitary matrix may be decomposed into a product of Givens rotation matrices, the control variables in the optimization problem now become the rotation angles θ_j. Figure 5 illustrates the frequency response of the first channel filter in a FIR paraunitary $\mathbf{H}(e^{j\omega})$ chosen to minimize the MSE with the ideal PCFB for the signal shown in Fig. 4. The brickwall response of the ideal channel filter is shown in blue.

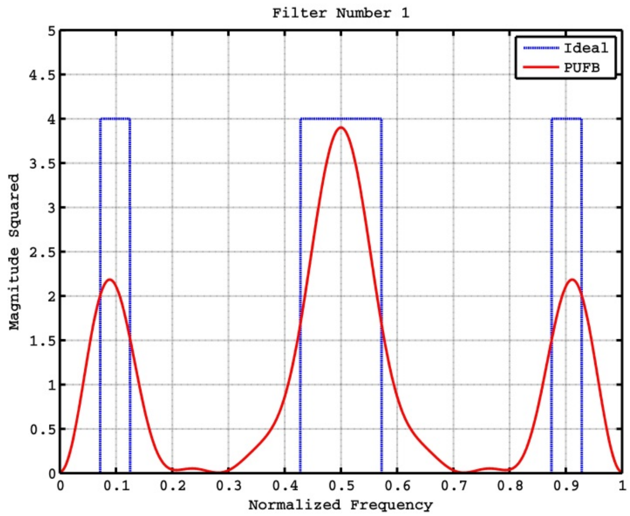

Figure 5. Paraunitary Approximation to Ideal PCFB – Channel 1

Figure 6 demonstrates the majorization property of a PCFB using the FIR paraunitary approximation $\mathbf{H}(e^{j\omega})$, with the number of channels M = 4. The plot overlays the cumulative sum of subband variances, expressed as percentages of the total signal energy, for the approximated PCFB and a DFTFB. As can be seen, the PCFB packs more signal energy into the lower subbands than the DFTFB.

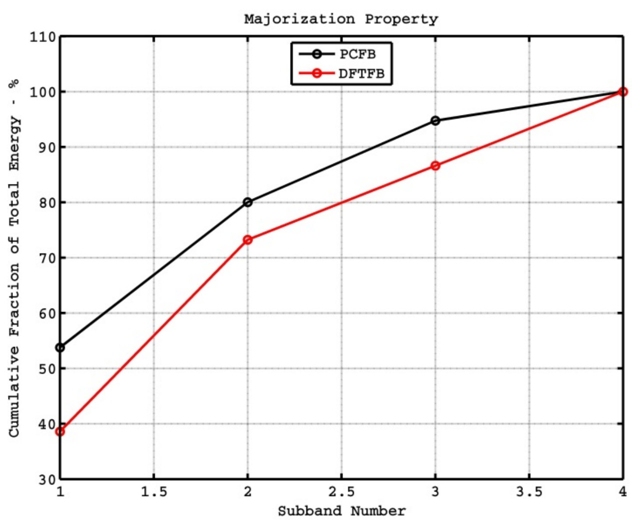

Figure 6. PCFB Majorization Property

VIII. PCFB Transmit Nulling Results

Figure 7 illustrates a point null achieved using an orthonormal approximation to an ideal PCFB for the signal in Fig. 4. The average null depth integrated over the signal bandwidth is −86.0 dBi.

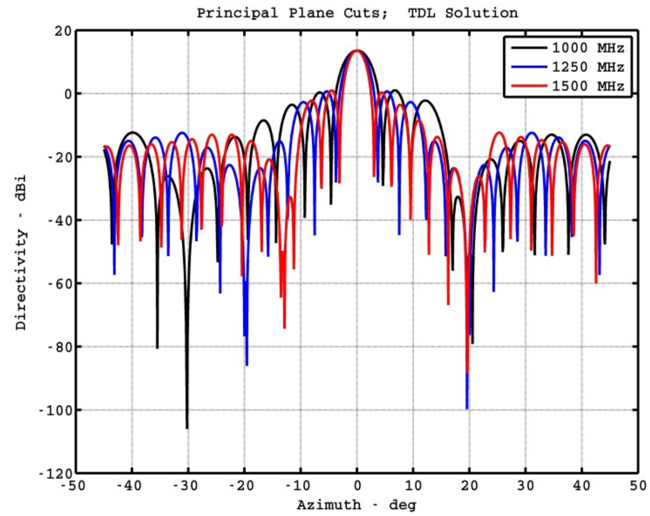

Figure 7. Point Null – PCFB Solution

A. Quantization Results

Consider the Effective Radiated Power (ERP) transmitted in the null direction θ_1 defined as $ERP = PG(\theta_1)$, where P is the transmitted signal power, and $G(\theta_1)$ is the antenna directivity. The ERP in the direction of the transmit null should be as low as possible. To illustrate the optimal behavior of the PCFB when the TDL coefficients are quantized in each subband, consider Fig. 8. This plot shows the cumulative probability distribution of ERP for a PCFB and a DFTFB when the bits allocated to each subband are $B_0 = 9$, $B_1 = 8$, $B_2 = 7$, and $B_3 = 6$. The average reduction in ERP for the PCFB compared to the DFTFB for this allocation of quantization bits is 1.94 dB. No particular strategy was used to allocate a precise number of quantization bits to each subband for the cases considered here. In general however, the preferred approach is to allocate more bits to the subbands with greater signal energy.

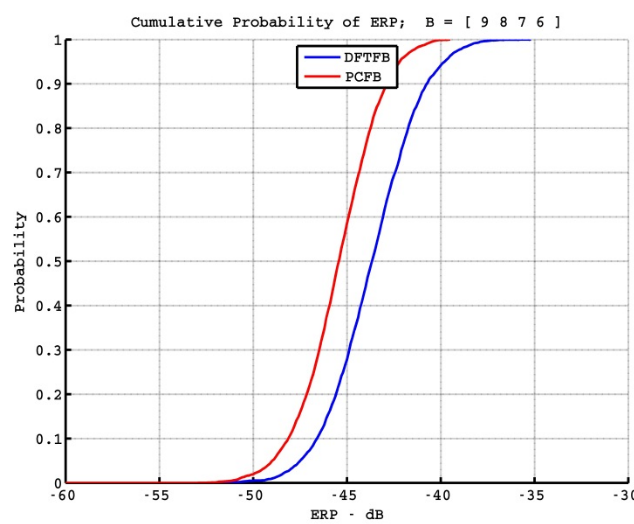

Figure 8. Cumulative Distribution of ERP in Null Direction; **B** = [9 8 7 6]

Figure 9 illustrates the case where $B_0 = 11$, $B_1 = 9$, $B_2 = 6$, and $B_3 = 6$. The average reduction in *ERP* for this case is 2.66 dB. Figure 10 illustrates the case with $B_0 = 11$, $B_1 = 9$, $B_2 = 7$, and $B_3 = 5$. The average reduction in *ERP* is 2.19 dB.

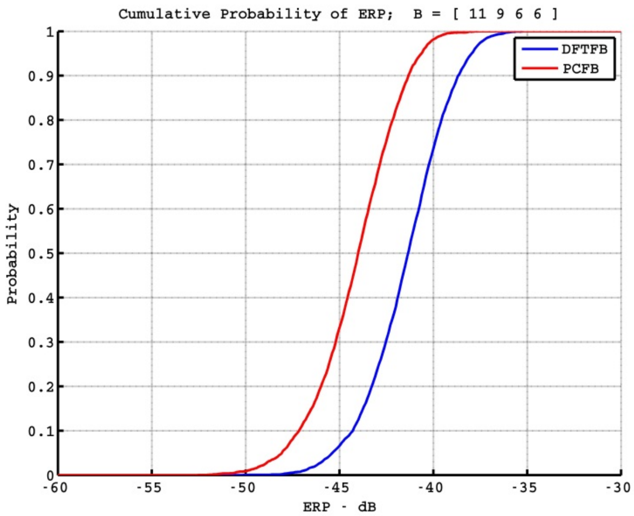

Figure 9. Cumulative Distribution of ERP in Null Direction; **B** = [11 9 6 6]

IX. CONCLUSIONS

To reduce the implementation complexity of a wideband Tapped Delay Line (TDL) array architecture, this paper proposed placing a filter bank behind each array element. The band partitioned architecture partitions the transmit signal into independent subbands and inserts a TDL in each subband to form a wideband spatial null. When the coefficients of the TDLs are subject to quantization errors, a band partitioning scheme based on Principal Component Filter Banks (PCFBs) is shown to be optimal and yield superior performance as compared to more conventional filter banks, such as the Discrete Fourier Transform Filter Bank (DFTFB).

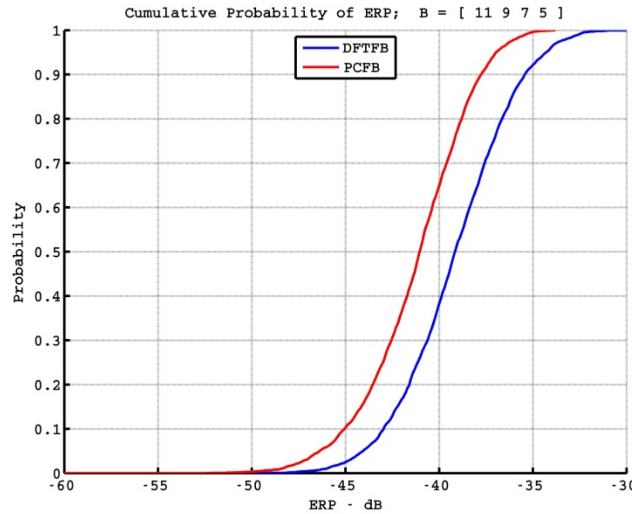

Figure 10. Cumulative Distribution of ERP in Null Direction; **B** = [11 9 7 5]

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